METHODICAL APPROACH TO PRICING AND VALUATION OF WEATHER DERIVATIVES IN NARIÑO COFFEE MARKET

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ABSTRACT

Agriculture in Colombia refers to all agricultural activities, essential to food feed, and fiber production, including all techniques for raising and processing livestock within the Republic of Colombia. The Colombian agricultural production has significant gaps in domestic and / or international human and animal sustenance needs. Particularly, Colombian coffee export sector has reached some significant achievement and is expected to develop. The climate often has a significant impact on agricultural production. Climate change has increased the frequency and intensity of catastrophic weather events that can cause serious financial damage to the farmers (see, e.g., Černý et al., 2013; Fukalová et al., 2014; Velecká et al., 2014). Actually, it is to be expected that weather fluctuations will increase in the future due to climate change (Pérez González and Yun, 2012). The main goal of this study is to construct the mathematical approach to pricing and valuation of Weather Derivatives in Nariño Coffee Market. The main goal of study is fourfold: 1) First, we begin our approach to brief introduction to weather derivative market. 2) We construct the mathematical for making bond with weather derivative financial derivatives (Weather options). 3) Then, we extend this approach to focus on valuation derivative with temperature payoff. 4) Finally, use the historical data from weather station in the Department of Nariño and construct the temperature model to evaluate the option pricing contracts. In addition, this paper ends with conclusion.

Key Words: Weather Derivatives, Temperature, Heating Cooling Degree Day (HDDs), Cooling Degree Days (CDDs), Option Contracts and Nariño Coffee Market

INTRODUCTION

A weather derivative is financial instruments, which are used to help a company or organization reduce the risks associated with adverse or unusual weather conditions. They work just like most other derivative contracts except the underlying asset (which is a weather condition such as rainfall, temperature or snowfall) has no value with which to price the derivative contract. Weather derivatives are used to hedge the risks of inclement weather conditions. They are measurable and essentially triggered by actual weather conditions making them a predictable form of risk management.

The new Post estimates that Colombian coffee production will increase to 13.6 million bags GBE in MY 2015/16 (October through September), slightly up from the previous forecast of 13.4 million bags GBE. Production is forecast to decrease in MY 2016/17 to 13.3 million bags GBE due to projected flooding conditions that may affect second harvest. The National Federation of Coffee Growers of Colombia (FEDECAFE) estimates that average coffee productivity has increased to 15 bags per hectare from 10 bags per hectare five years ago. This is a direct result of the replanting program and a reduction in the average age of coffee trees from 15 to 7 years. The rust resistant variety replanting program and minimal impacts of El Nino weather phenomena on the main harvest (October to December) have helped to maintain strong productivity during the first seven months of MY 2015/16, up 15% from the same time period a year before. Domestic prices are managed by FEDECAFE and based on the daily quote of the New York Coffee, Sugar and Cocoa Exchange (NYCSCE) less estimated costs for internal transport and administration. Since second semester 2014, coffee prices rose levels above the Government of Colombia (GOC) Protection for the Income of Farmers (PIC) subsidy program trigger price. The increase of internal price was driven by devaluation of Colombian peso and international commodity markets behaviour after reducing uncertainty about the impact of drought in world production. The GOC continues to suspend the PIC program. In MY 2015/16, Post estimates that exports will reach 12.3 million bags GBE paralleling the increase in production. Exports are forecast to remain unchanged at 12.3 million bags GBE for MY 2016/17.

1. WEATHER DERIVATIVE MARKET

Weather derivatives are usually structured as swaps, futures, and options based on different underlying weather indices. Here, introduce some indexes frequently used on the weather derivatives market, which is the underlying of the temperature.

The main temperature indices are so called Cooling Degree Days (CDDs), Heating Degree Days (HDDs) and Cumulative Average Temperature (CAT).

Given a weather station, let us note by $T_i^{\max}$ and $T_i^{\min}$, respectively, the maximum and the minimum temperature (generally in degree Celsius) measured in one day i.

$$T_i = \frac{T_i^{\max} + T_i^{\min}}{2} \qquad (1)$$

As above mentioned, one important underlying variables for weather derivatives is the degree – day. For a given location, the degree – day is the temperature value of the different between the temperature of given day and a temperature threshold.

This quantity is defined below.

Let define T_i as the mean temperature of a day i. We define Heating Degree Days (HDD_i - measure of cold in winder) and Cooling Degree Days (CDD_i - measure of heat in summer), generated on that day as

$$HDD_i = \max\left(T_{ref} - T_i, 0\right) \qquad (2)$$

$$CDD_i = \max\left(T_i - T_{ref}, 0\right) \qquad (3)$$

Where T_{ref} is a is the reference temperature (in general between 18^0 C and 20^0 predetermined temperature level and T_i is the average temperature calculated as in (1) for a given day i.

From the Eq. (2), we can write cumulative HDD (CHDD):

$$CHDD = \sum_{i=1}^{N} HDD_i \qquad (4)$$

Where HDD_i is calculated as in (2) and *N* is the time horizon, which is generally a month or a season.

The CAT index is for main cities in a country. For a measurement period $[T_1, T_2], T_1 < T_2$ the CDD index is measuring the demand for cooling and is defined as the cumulative amount a threshold *c*.

Mathematically we expressed it as

$$\sum_{t=T_1}^{T_2} \max[T_i - c, 0] \qquad (5)$$

with T_i being the daily average temperature on day *i*, and the average is computed as the mean of the daily maximum and minimum observations. The threshold *c* is in the market given as 18^0C, or 65^0F and is the trigger point for when air-conditioning is switched on. The measurement periods are set to weeks, months, or seasons consisting of two or more months, within the warmer parts of the year. A futures written on the CDD pays out an amount of money to the buyer proportional to the index, in US dollars for the American market, and in Euro for the European one.

An HDD index is defined similarly to the CDD as the cumulative amount of temperatures below a threshold *c* over a measurement period $[T_1, T_2]$ that is,

$$\sum_{t=T_1}^{T_2} \max[c - T_i, 0] \qquad (6)$$

The index reflects the demand for heating at a certain location, and measurement periods are typically weeks, months, or seasons in the cold period (winter).

The CAT index is simply the accumulated average temperature over the measurement period defined as

$$\sum_{t=T_1}^{T_2} T_i \qquad (7)$$

This index is used for country cities at Chicago Mercantile Exchange (CME). CME also operates with a daily average index called the Pacific Rim measured in several cities. The Pacific Rim is simply the average of daily temperatures over a measurement period, with the daily temperature defined as the mean of the hourly-observed temperatures. Beyond the obvious initial applications of weather derivatives to hedging energy risk, the market has expanded to address a wide array of weather risks faced by numerous other industry sectors. A U.S. Department of Commerce estimate indicates that more than $1 trillion of U.S. economic activity is exposed to the weather, and transactions over the past several years have provided weather protection to companies in sectors as diverse as entertainment, retail, agriculture, and construction. A sampling of weather risks faced by various industries is presented in the table below.

Table 1 - Illustrative Links between Weather and Financial Risk

Risk Holder	**Weather Type**	**Risk**
Energy Industry	Temperature	Lower sales during warm winters or cool summers
Energy Consumers	Temperature	Higher heating/cooling costs during cold winters and hot summers
Beverage Producers	Temperature	Lower sales during cool summers
Building Material Companies	Temperature/Snowfall	Lower sales during severe winters (construction sites shut down)
Construction Companies	Temperature/Snowfall	Delays in meeting schedules during periods of poor weather
Ski Resorts	Snowfall	Lower revenue during winters with below-average snowfall
Agricultural Industry	Temperature/Snowfall	Significant crop losses due to extreme temperatures or rainfall
Municipal Governments	Snowfall	Higher snow removal costs during winters with above-average snowfall
Road Salt Companies	Snowfall	Lower revenues during low snowfall winters
Hydro-electric power generation	Precipitation	Lower revenue during periods of drought

For mathematical convenience, we will use integration rather than summation and define the three indices CDD, HDD, and CAT over a measurement period $[T_1, T_2]$ by

$$CAT(T_1, T_2) = \int_{T_1}^{T_2} T_i dt \tag{8}$$

$$CDD(T_1, T_2) = \int_{T_1}^{T_2} \max[T_i - c, 0] dt \tag{9}$$

$$HDD(T_1, T_2) = \int_{T_1}^{T_2} \max[c - T_i, 0] dt \tag{10}$$

2. WEATHER OPTION CONTRACTS

The most common of these contracts come in the form of either Heating Degree Days (HDDs) or Cooling Degree Days (CDDs) contracts. The payoff of these contracts is based on the cumulative in the daily temperature relative to 18^0C (about 64^0F) over a fixed period such as a month. The fixed level of 18^0C is the temperature at which the energy sector believes little heating or cooling occurs in households. The buyer from a positive payoff if cumulative temperature is below or above a specified level.

Outside the CME, there are a number of different contracts traded on the OTC market. One common type of contract is the option. There are two types of options, calls and puts. The buyer of a HDD call, for example, pays the seller a premium at the beginning of the contract. In return, if the number of HDDs for the contract period is greater than the predetermined strike level the buyer will receive a payoff. The size of the payoff is determined by the strike and the tick size. The tick size is the amount of money that the holder of the call receives for each degree-day above the strike level for the period. Often the option has a cap on the maximum payoff unlike, for example, traditional options on stocks.

A generic weather option can be formulated by specifying the following parameters: - The contract type (call or put), The contract period (e.g. January 2015), The underlying index

(HDD or CDD), An official weather station from which the temperature data are obtained, the strike level, the tick size and the maximum payoff (if there is any).

To find a formula for the payoff of an option, let *K* denote the strike level and α the tick size. Let the contract period consist of *n* days. Then the number of HDDs and CDDs for that period are

$$H_n = \sum_{i=1}^{n} HDD_i, \tag{11}$$

$$C_n = \sum_{i=1}^{n} CDD_i, \tag{12}$$

respectively.

Now we can write the payoff of an uncapped HDD call as

$$\chi = \alpha \max\{H_n - K, 0\} \tag{13}$$

The payoffs for similar contracts like HDD puts and CDD calls/puts are defined in the same way.

HDD and CDD are by their nature quantitative indices derived from daily temperature measurements, which are designed to reflect the demand for energy needed to heat or cool houses and factories. The idea of HDD consists in the fact that heating is usually required when temperature drops below some reference level and thus energy expenditure is needed. Heating or cooling requirements for a given subject at a specific geographical location are commonly considered directly proportional to the number of degree-days. For detailed overview of weather indices in use see also Barrieu and Scaillet (2010) or Cao et al. (2004b).

HDD is defined as the number of degrees by which the daily average temperature is below some base temperature, while CDD express the number of degrees by which the daily average temperature is above this value. Mathematically expressed, daily HDD and CDD structures look as follows:

Eq (1) we can write

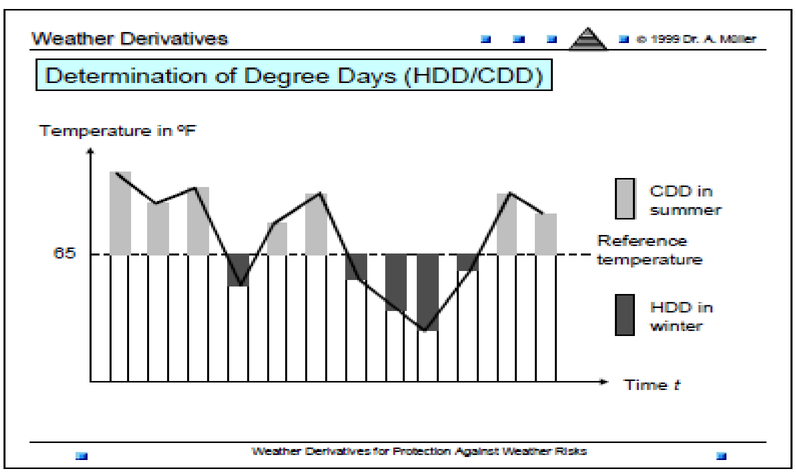
$$HDD = \max\left(0, T_{base} - T_{Daily-Average}\right) \quad \forall T(t) < 65 \tag{14}$$

And eq (2) we can write

$$CDD = \max\left(0, T_{Daily-Average} - T_{base}\right) \quad \forall T(t) > 65 \qquad (15)$$

The base approach in the energy sector, which has been previously used in numerous studies (see e.g. Cao et al. 2004a, Alaton et al. 2005 or Clemens et al. 2008), is to apply 18°C (= 65 degrees Fahrenheit) for HDD and 24°C (= 75 degrees Fahrenheit) for CDD as the base temperature. Daily HDD and CDD are consequently accumulated over a period of time (usually of few months or whole season) that serves as an indicator of heating or cooling requirements for this period.

Figure 1: Heating Degree Days (HDDs) and Cooling Degree Days (CDDs)



Considering the interest of natural gas companies, increase in HDD corresponds with decreasing temperature, which is reflected in higher natural gas consumption. Since weather derivatives are applied for hedging of weather related risks over a long horizon, production of aggregate indices is needed. Campbell and Diebold (2005) demonstrate the importance of a cumulative HDD index (see equation (24)), which includes both nonlinear transformation of daily average temperature into HDD as well as further aggregation of daily indices because:

- Weather derivatives are typically written on a cumulative sum of weather related outcomes
- November-March HDD contract is one of the most actively traded weather-related contracts and is also of a substantial interest to end users of weather models

$$Cumulative \quad HDD = \sum_{t=1}^{n} HDD_t = \sum_{t=1}^{n} \max\left(T_{base} - T_t, 0\right) \qquad (16)$$

The indices described above define how weather variability is encapsulated for the purposes of a weather derivative contract. The contract is then financially settled using the measured value of the index as the input to a pay-off function. This function defines precisely who should pay what to whom at the end of the contract. Any function could be used as a pay-off function, but in practice only a small number of simple structures, with straightforward economic purposes, are common. We will consider the payoff for each of these structures from the point of view of the buyer of the contract, who is said to take the 'long' position. The seller of the contract, who takes the 'short' position, will have exactly the opposite pay-off.

Figure 2: The payoff function for the option contract

As we have already stated, the most commonly used weather derivatives are options, especially calls and puts as well as their various combinations, e.g. collars. With regard to intentions of particular hedges, a company generally decides on various option types shown in Table. Beside various purposes of particular options, this table introduces also simple drafts of payoffs that are generally based on the difference between the exercise and actual level of a weather index.

<u>Table 2 – Temperature Options</u>

Option type	Protection against	Exercise when	Payout
HDD Call	Overly Cold Winters	HDD > Strike	Tick*(HDD-Strike)
HDD Put	Overly Warm Winters	HDD < Strike	Tick*(Strike-HDD)
CDD Call	Overly Hot Summers	CDD > Strike	Tick*(CDD-Strike)
CDD Put	Overly Cold Summers	CDD < Strike	Tick*(Strike-CDD)

As we are primarily interested in fluctuations of natural gas consumption during winters, CDD are not considered in this paper as this index corresponds especially to requirements for cooling energy. In spite of wide flexibility available in designing weather derivatives, basic attributes are common for the majority of contracts. Therefore, several basic features have to be specified to determine the payoff from a HDD option. On the day following the end of a contract period, the payoff from an option may be computed in compliance with following equations.

For Call option $V = \min(\max\{0, (HDD - X)\} * tick, cap)$ (17)

And for Put option $V = \min(\max\{0, (X - HDD)\} * tick, cap)$ (18)

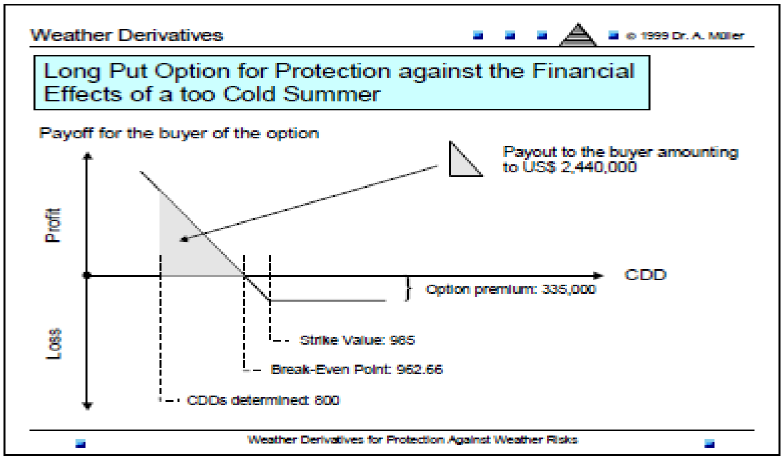
Where *X* means the strike level, HDD is an aggregate level of the index, Tick is the payment per one HDD and Cap is the maximum payment from an option.

Table 3 - Measure CDD

City	State	Start	End	Deal	Strike Value *	Tick Size in US$	Limit in US $	Price in US$
Cincinnati	Ohio	1/5/09	31/10/09	CDD Put	985	15, 000	3 Mil	335, 000

*Measure in CDD, determine the exercising of the option

Figure 3: Exemplary payoff diagram for a Long Put



3. PRICING PRINCIPLES

The theory in terms of weather derivative pricing is still extremely sparse with no widely satisfactory formula that can be used by the growing number of practitioners in the marketplace. The current pricing methodologies can be broadly put into two groups; analytical solutions or numerical solutions. Whilst standard equity options have the famous Black-Scholes equation to provide practitioners with a reliable pricing basis, their weather-based counterparts have no agreeable equivalent to the BS framework and generally require a numerical approach.

A quick revision of standard option pricing theory is helpful when investigating its extensions later. Arithmetic Brownian motion is commonly represented by the following stochastic differential equation:

$$dX_t = \mu.dt + \sigma.dW_t \tag{19}$$

Whilst this might be suitable for modelling biological processes, Geometric Brownian motion (GBM) is the process that is generally used to model financial variables such as stock and commodity prices. Its necessity arises out of the fact that a log function does not permit negative values, essential when modelling asset prices. GBM is described by the following stochastic differential equation:

$$\frac{dX_t}{X_t} = \mu.dt + \sigma.dW_t \tag{20}$$

This can be reduced to arithmetic Brownian Motion via the substitution into the above equation $y = F(X_t) = \log X_t$ of and the use of Ito's formula, to give:

$$dF = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma.dW_t \tag{21}$$

which with an initial condition, X_0, has a solution given by:

$$F(X_t) = \log x_0 + \left(\mu - \frac{1}{2}\sigma^2\right)(t - t_0) + \sigma W_{t-t_0} \tag{22}$$

A solution for the process X_t is then found by reversing the substitution (exponentiation), Hence we arrive at our distribution for the initial GBM process:

$$X_t = X_0 e^{\left[\left(\mu - \frac{1}{2}\sigma^2\right)(t-t_0) + \sigma W_{t-t_0}\right]} \tag{23}$$

From here Black and Scholes use their now famous hedging method to derive a partial differential equation (i.e. no longer a stochastic equation) for the dynamics of the option price that is based on the Brownian motion of equation (20). The Black-Scholes PDE is:

$$\frac{\partial V_t}{\partial t} + rV - ry\frac{\partial V}{\partial y} - \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 V}{\partial y^2} \tag{24}$$

Where V_t represents the payoff of the option contract. This differential equation can then be solved to obtain the explicit Black-Scholes formula. Alternatively, a martingale approach can be adopted via equation (23), i.e. seeking a solution to the equation:

$$V_t = \mathbf{E}_Q\left[X_0 e^{\left[\left(\mu-\frac{1}{2}\sigma^2\right)(t-t_0)+\sigma W_{t-t_0}\right]} \middle| F_t \right] \qquad (25)$$

The key difference to modelling weather-based variables when compared with a general financial variable, such as a stock price, is that most weather components exhibit some degree of mean reversion. A mean reverting process is one in which the drift component of the stochastic differential equation (19) always acts in a direction that opposes the current displacement from the mean process, in much the same way as a spring acts on a weight.

This concept is utilized frequently in the modelling of interest rates, which like many weather variables are at least partially mean reverting in that do not rise or fall without bound. The Vasicek(1979) model of forward interest rates is based on this mean reverting approach as well as other well-known interest rate model such as the Hull-White model and the Cox-Ingersoll-Ross model.

The mean reversion component is deterministic and is an extension of the drift term, μ

$$\frac{dX_t}{dt}\alpha-\left(X_t-\overline{X}\right) \qquad (26)$$

where $\overline{X}$ represents the mean process. In this way the drift will always act in a way so as to bring the process closer to its mean. A proportionality constant is now required, called the mean-reversion parameter, which is a measure of the restoration force acting on the process dynamics. It is akin to the spring constant k, for all those who can still recall high school physics classes.

$$\frac{dX_t}{dt}=-\gamma\left(X_t-\overline{X}\right) \qquad (27)$$

Substituting this into the standard Brownian motion dynamics, equation (19), we get:

$$dX_t=\gamma\left(\overline{X}-X_t\right)dt+\sigma dW_t \qquad (28)$$

Note that this process will no longer be gBm and that negative values are permitted by equation (28). The probability that negative values will occur depends on the mean process level as well as the strength of the mean reversion. For a strongly reversionary process who's mean is significantly above zero it is highly unlikely that the process would ever go negative. The Ornstein-Uhlenbeck process is the name given to this modified Brownian motion. It has since been shown in Dornier and Queruel [2002] that the process given in equation (26) does not actually revert to its mean when temperature is used as the variable. This is due to the fact that the mean process that the equation is reverting to, $\overline{X}$, is not constant. To overcome this an extra term (the time derivative of the mean process) is required to be added to the drift component of the stochastic equation above. Hence we get:

$$dX_t = \left[\gamma\left(\overline{X} - X_t\right) + \frac{d\overline{X}}{dt}\right]dt + \sigma dW_t \qquad (29)$$

This representation is now mean reverting in the long-run, in other words: $\mathrm{E}\left[X_t\right] = \overline{X}$.

Another advantage of including this term is that equation (29) can now be solved by the traditional integrating factor method. Multiplying through by $e^{\gamma s}$ we obtain:

$$e^{\gamma s} dX_s = e^{\gamma s}\gamma\left(\overline{X} - X\right)dt + e^{\gamma s} d\overline{X}_s = e^{\gamma s}\sigma dW_s \qquad (30)$$

Now the left hand side of the above expression is just the differential of a product, i.e:

$$e^{\gamma s} dX_s - e^{\gamma s}\gamma\left(\overline{X} - X\right)dt + e^{\gamma s} d\overline{X}_s = de^{\gamma s}\left(X_s - \overline{X}_s\right)$$

$$d\left[e^{\gamma s}\left(X_s - \overline{X}_s\right)\right] = \sigma\int_0^1 e^{\gamma s} dW_s$$

Note that this would not have been possible without the extra term being added to equation (29) to make it properly mean reverting. After rearranging this expression, we obtain the solution to the stochastic process:

The seminal paper of Black and Scholes [1977] provided an analytical framework for the pricing of contingent claims and in particular options, however the Black Scholes (BS) formula relies on some fairly stringent assumptions. Most importantly it is assumed that the underlying process is driven by gBm as given above by equation (19). Most empirical studies show that in fact asset returns are strongly leptokurtic (More concentrated in the middle and 'fat-tailed'), such as Fama (1965).

As well as this, with respect to weather derivatives, there is generally no underlying process that is actively traded (i.e. people don't trade degrees …. yet) and as such the BS framework has no method of hedging the derivative in order to derive an analytical solution. For these reasons a standard BS approach is not applicable and other more indirect methods must be pursued.

Many common weather derivative contracts have as their underlying index, an average of some statistic over a period of time. For example, many temperature - based derivatives will have a payoff that is determined by reference to the average temperature over a week or a month. These types of options are generally referred to as 'Asian' options and are common in both fixed-interest and equity derivative markets. The averaging can be done on a variety of time frames however it is generally the daily temperature that is averaged. For example, a call option on the monthly average temperature would have payoff of the form:

$$V_t = \max\left\{0, \left(\frac{\sum_{i=1}^{n} T_i}{n} - K\right)\right\}.Tick \qquad (31)$$

Where there are n days in the month. Analogies to the BS partial differential equation can be derived for the varieties of 'Asian' options that exist however, as outlined above, we still have no underlying asset with which to perform the required hedge. These types of contracts are particularly appropriate for rainfall-based derivatives whose discreetness can be smoothed by averaging over a larger period.

Several authors have decided to proceed with the BS framework ignore the problems of the underlying assumptions or have attempted to alter the BS approach to accommodate these deficiencies. The most serious assumption that must be relaxed is that there is no underlying asset to base the derivative price around. Jewson et. al [2003] has suggested that an alternative BS formula can be derived akin to the derivation of an option on a futures contract. To overcome the fact that there does not exist an underlying, traded asset, Jewson creates a hypothetical forward weather index that is used as the underlying asset which the BS framework can use as a hedge to enable the price contingent claim.

The theory is based on the Black (76) model where a future contract is used as the hedge when deriving the partial differential equation that governs its motion. To illuminate, let us assume that the futures price process is governed by the 'cash-carry' relationship:

$$Y_t = X_t e^{r(T-t)} \tag{32}$$

By using ito´s formula when substituting this relationship into the standard GBM form equation (19) we get the altered stochastic differential equation:

$$dY_t = y[(\mu - r)dt + \sigma dW_t] \tag{33}$$

Following the same hedging procedure as used in deriving the Black Scholes partial differential equation we arrive at the following relationship for the dynamics of option on futures contract:

$$\frac{dV_t}{dt} = rV - \frac{1}{2}\sigma^2 y^2 \frac{d^2V}{dy^2} \tag{34}$$

If we compare this relation with equation (24) we can see that the term, $-ry\frac{\partial V}{\partial y}$ .is removed from the right hand side of the equation. This is the same equation one would get if calculating the price of an option on a dividend paying stock where the dividend yield was equal to the risk free rate. Hence by using notation similar to Buchen [2002] we can write the option price over a futures contract as:

$$V(y,t) = BS(ye^{-r\tau}, t, r, \sigma) = e^{-r\tau} BS(y, t, 0, \sigma) \tag{35}$$

where $BS(x,t,r,\sigma)$ represents the 'standard' Black Scholes pricing formula. This formula can then be used, after appropriate modelling of the futures price process, to calculate the premium applicable to a range of weather based option contracts.

This is a typical actuarial approach adopted to price a contingency where no assumptions are required to be made as to the nature of the process on which the contingency relies.

A typical ‘burn’ analysis seeks to answer the question:” What would be the return from the contract had I purchased it each year for the last x years?” Generally, an arithmetic average is then taken of the results.

For example, to price a February HDD call with exercise of 100, simply find what the financial return would have been for each of the February months in the historical data set, with the appropriate indexing of the exercise value in order to standardize the temperatures over time.

4. TEMPERATURE MODELING

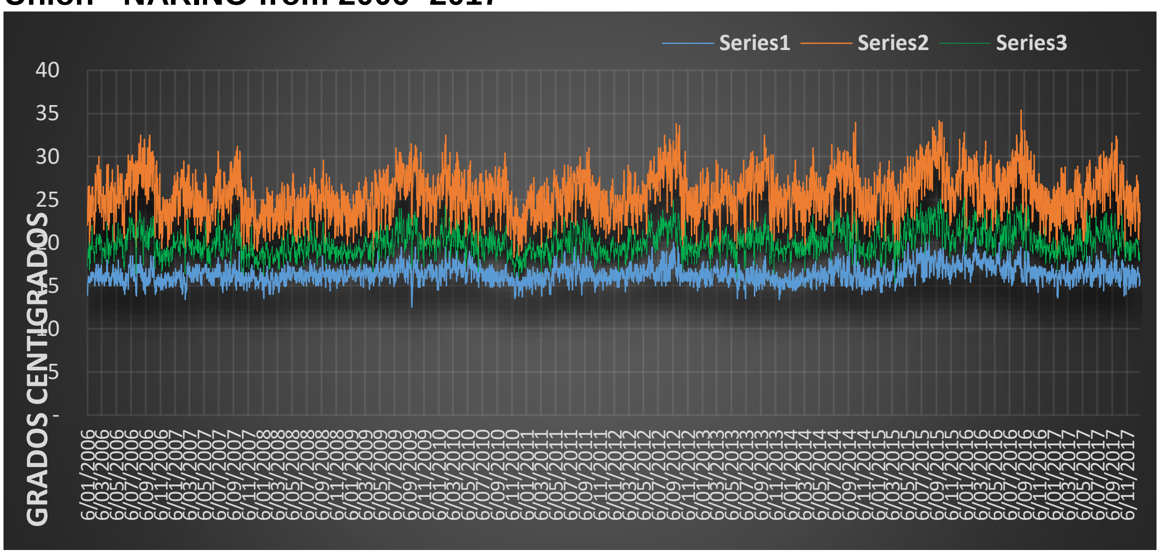
Compared with rainfall, the analysis of temperature has received significant attention from the literature in recent years. Primarily this is due to the fact that the majority of traded contracts in the weather derivatives market are temperature based, generally related to energy supply or demand. More recently the search for a reliable statistical model for temperature dynamics has been intensified by the need to provide sound evidence for the impacts of human interaction on the planet. The temperature model that is developed in the next sections follows a combination of approaches used by Benth et al [2002] and Alaton [2002] and has been applied to a range of weather station recordings.

There is not a market for weather derivatives in Colombia. This is attributed to the fact that it is unclear whether and to what extent weather derivatives are a useful instrument of risk management. Since the most common underlying variable is temperature, only the temperature-based derivatives will be considered in this paper. A simple pricing formula for an option contract depending on temperature index HDD will be suggested. Agricultural application of weather derivatives to risk management will be indicated.

It is important to evaluate how climate has varied and changed in the past. The monthly mean historical temperature data can be mapped to show the baseline climate and seasonality by month, for specific years, and temperature. The chart below shows mean historical monthly temperature for department of NARIÑO coffee market during the time period 2006 - 2017.

The data were collected for the studied at Municipally la Union - NARIÑO coffee market. Our methodology can be applied to different cities. The time period consists of 12 years (from 2006 to 2012) with 4383 (daily) observations. The data consists of monthly mean minimum and maximum temperatures from the weather station in the mentioned area. Historical temperature observations were obtained from the National climatic data center. In Figure 4 is shown the daily average temperature in the studied area.

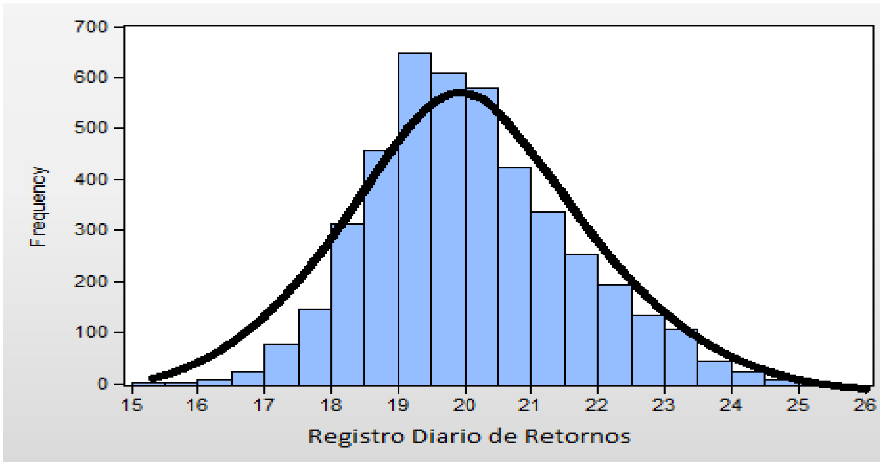
Figure 4 - Graph for the Daily Average Temperature development in Municipally la Union - NARIÑO from 2006 -2017



We use the burn analysis based on historical data from the studied area to compute the price of the option with payoff depending on temperature. The price of the option depends on the temperature index and the strike price.

A histogram, corresponding to the above data is placed at Fig 5.2. It is clear from the data's that the assumption normality daily log returns. And we obtained the mean and standard deviation is 20.03 and 1.4829 respectively.

Figure 5 – Histogram of daily Log Returns from 2006 – 2017



Further the histograms classified into twelve class with each year and average temperature and the real data from the GBM and also we performed the mean and variance of the given dates.

The results of the analysis are tabulated below.

Table 4 - Mean and Median for the average daily temperature from 2006 – 2017

Data's	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**
Media	19.85	19.58	19.14	20.08	19.75	19.63	20.19	19.91	20.07	21.19	20.90	20.19
S.D	1.479	1.286	1.052	1.378	1.453	1.25	1.517	1.161	1.419	1.733	1.458	1.306
N	365	365	366	365	365	365	366	365	365	365	366	365

The histogram of each class intervals are placed in fig 6.

2006

2007

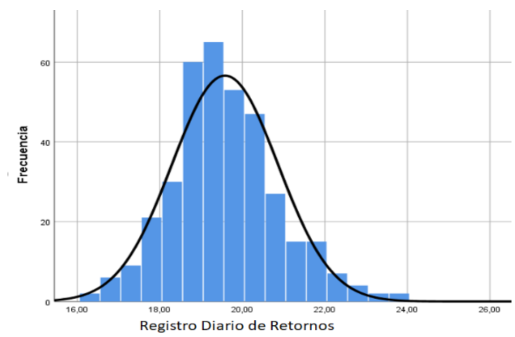
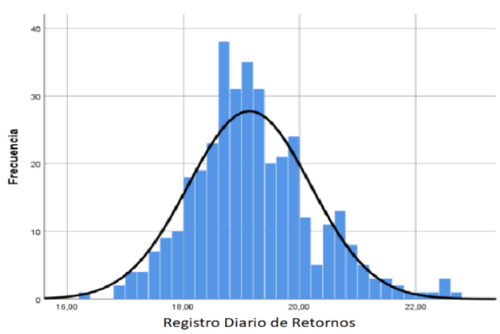


Fig 6 (A)

Fig 6(B)

2008

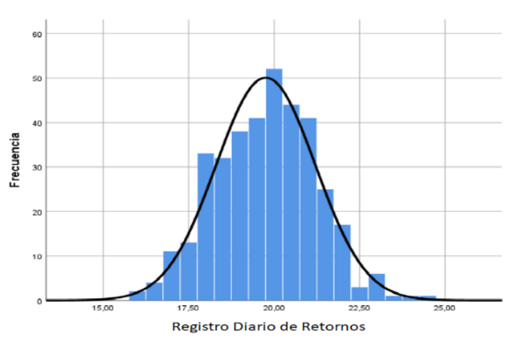


2009

Fig 6 (C)

Fig 6 (D)

2010



2011

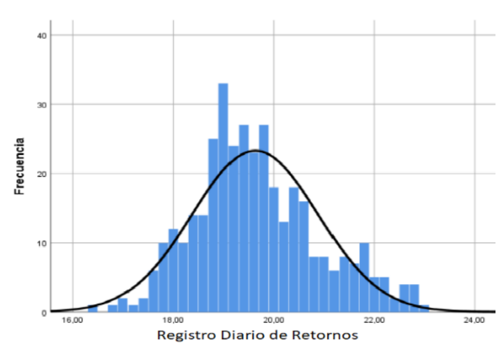
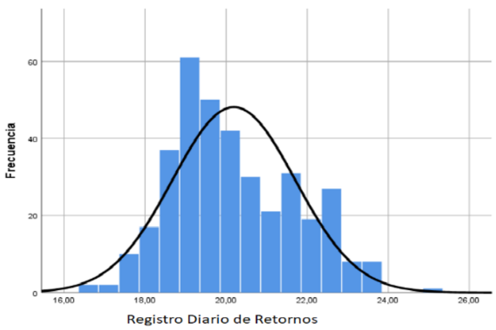


Fig 6 (E)

Fig 6 (F)

2012

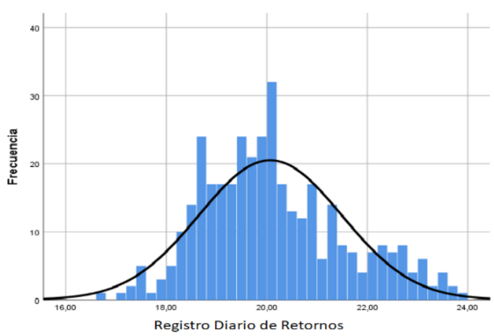


2013

Fig 6 (G)

Fig 6 (H)

2014



2015

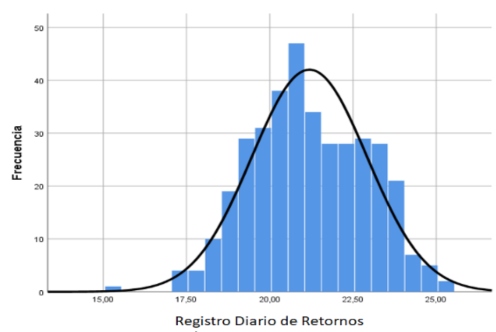


Fig 6 (I)

Fig 6 (J)

2016

2017

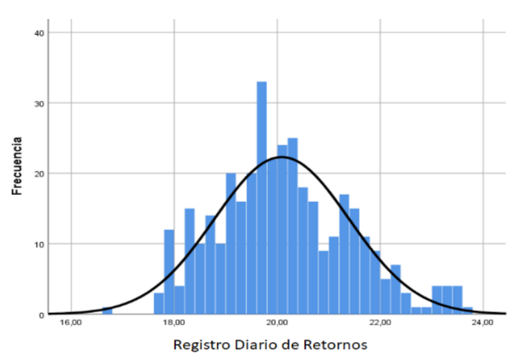


Fig 6 (K)

Fig 6 (L)

Now, we use the burn analysis based on historical data from our studied data to compute the price of the option with payoff.

Based on commonly used temperature index HDD (heating degree-days index) presented by Zapranis and Alexandridis (2009), Benth and Benth (2011) and many others the options are priced. The daily value of the HDD index is:

$$Daily \quad HDD(t) = Max[18 - T(t), 0] \qquad (36)$$

where *T(t)* is the daily average temperature. The daily average temperature is calculated by averaging each day's maximum and minimum temperature from midnight-to-midnight. To illustrate the concept of HDD, suppose that on a given winter day the high temperature was 25.30 degrees and the low temperature was 15.30 degree. This weather results in a daily average temperature of 10 degrees with 40.6 HDD for this day. The lower is the temperature, the higher is the daily HDD. Weather options are written on the cumulative HDD over a specified period. The annual payoff is calculated as:

$$Annual \quad Payoff = Annual \; Average \quad HDD \times Tick \quad Leval \qquad (37)$$

The owner of a call option obtains the payoff if the HDD index value is higher than the strike price. Assuming the put option owner, the payoff is paid if the strike price is higher than the HDD index value. Let us denote the annual average payoff μ, the standard deviation calculated from the annual payoffs σ, the risk-free interest rate r, the maturity date T, the relation to the risk α. The long and short option price is:

$$\begin{aligned} Long &= e^{-rt}(\mu + \alpha.\sigma) \\ Short &= e^{-rt}(\mu - \alpha.\sigma) \end{aligned} \qquad (38)$$

The strike prices X calculated according to the papers (Garcia and Sturzenegger, 2001; Platen and West, 2004; Roustant et al., 2003) have the following forms:

$$\begin{aligned} A &: X = Annual \quad Average \quad HDD; \\ B &: X = Annual \quad Average \quad HDD - \frac{S.D \, of \; Annual \, HDD}{2} \\ B &: X = Annual \quad Average \quad HDD + \frac{S.D \, of \; Annual \, HDD}{2} \end{aligned} \qquad (39)$$

5. RESULT AND DISCUSSION

In pricing we consider the following parameters.

1. Risk- free interest rate is AAA – rated Colombian government bond yield for 30 years' maturity relating to the historical data.
2. The time of HDD option contract conclusion is 31st of December 2017 and the contract is for annual temperature events,
3. 15% relation to the risk.

The partial results used in our analysis are in Table 5.

Strike Price in USD	Price of the Option in USD
2225	4.93
2351	4.7
2465	4.45

Source: Author´s own Calculations

Table 5.2 - Call/Put Option Pricing

Weather risk for producers in coffee business refers to the uncertainty about the expected weather conditions. If a farmer wanted to have protection from an extremely cold winter he could buy HDD call options or HDD put options, for an extremely hot winter. For example, if a wheat farmer wanted to use options to hedge his cold weather risk, he could buy HDD call options for some city at some strike. The buyer of a weather option has theoretically unlimited profit potential on the upside, while only risking the premium paid for the option on the downside. This limited downside risk is transferred to the writer of the option, who accepts potentially unlimited downside risk in return for receiving the option premium at the time of the sale. The HDD options are European style, which means that they cannot be exercised before their respective maturity dates. The following hedging strategies based on HDD index are examples of how to manage weather risk using option contracts.

Let construct a call option on HDD index with various strike prices. We assume that the call option on the HDD index will buy a farmer whose profits are affected by low temperatures.

HDD Index Value Scenario	Payoff
HDD<X	$-c_B$
HDD≥X	HDD-X-c_B

Source: The author's processing

This is explaining the Payoff from the buying call option

On 31 December 2017 we will buy the call option on HDD index with the strike price X = 2225. The income function for weather option 1 (WO1) is:

$$P_1(HDD) = \begin{cases} -4.93 & \text{if } HDD < 2225 \\ HDD - 2229.93 & \text{if } \geq 2225 \end{cases}$$

The income function for weather option 2 (WO2) with the strike price X=2351 has the following form:

$$P_2(HDD) = \begin{cases} -4.7 & \text{if } HDD < 2351 \\ HDD - 2355.7 & \text{if } \geq 2351 \end{cases}$$

and for weather option 3 (WO3) with the strike price X = 2465 is:

$$P_3(HDD) = \begin{cases} -4.45 & \text{if } HDD < 2465 \\ HDD - 2469.45 & \text{if } \geq 2465 \end{cases}$$

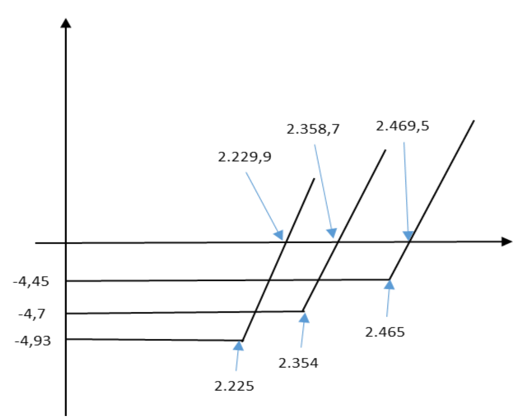
For an option buyer, the premium represents the maximum cost or amount that can be lost, since the option buyer is limited only to the initial investment.

The comparison of incomes of WO1, WO2 and WO3 at various development of HDD index value at the maturity time of options is shown in Fig. 2. It can be seen, but also be calculated exactly using income function presented above, that:

- the weather option 3 (with a strike price 3437.88 calculated using formula C), if the HDD index value at the maturity date of options is lower than USD 2230;
- the weather option 1 (with a strike price 2225 calculated using formula (B)), if the HDD index value at the maturity date of options is higher than USD 2229.93.

The lower the strike price, the higher the income from the call option. The income is unlimited; the loss is limited by the option premium. The weather option 1 ensures the highest profit. The cost of this benefit is the highest option premium. This hedging variant is available to the farmer with a higher degree of risk aversion. Low-risk-aversion farmer will prefer the weather option 3. The weather option 2 is the most suitable hedging strategy for the farmer with a neutral risk aversion. It should be noted that, only the weather option with the strike price USD 2351 is used for further analysis.

Figure 7 – Graph Illustrated payoff from the weather Options 1, 2 and 3 proposed for Nariño: The author´s Processing



The Long Put strategy is a simple bearish strategy. By buying put option the buyer hedge against a temperature increase. The following table lists the payoff from buying put option.

Payoff from the buying put option

HDD Index Value Scenario	Payoff
HDD<X	-HDD+X-PB
HDD≥X	-PB

Source: The author's processing

This Table is explaining the Payoff from the buying put option

Assume the put option on HDD index with the strike price X = 2225. The income function of weather option 4 (WO4) is expressed by the formula:

$$P_4(HDD) = \begin{cases} HDD + 2220.07 & if\ HDD < 2225 \\ -4.93 & if \geq 2225 \end{cases}$$

The income function for weather option 5 (WO5) with the strike price X=2351 has the following form:

$$P_5(HDD) = \begin{cases} HDD + 2346.3 & if\ HDD < 2351 \\ -4.7 & if \geq 2351 \end{cases}$$

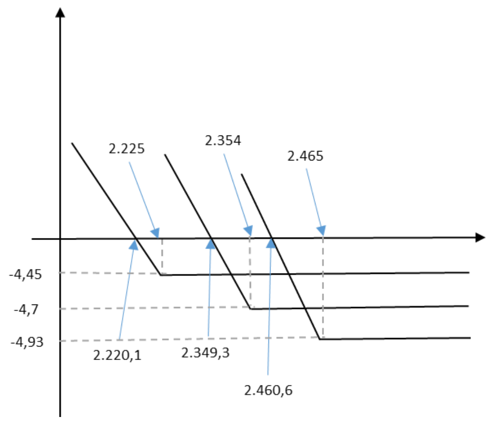
and for weather option 6 (WO6) with the strike price X = 2465 is:

$$P_6(HDD) = \begin{cases} HDD + 2460.55 & if\ HDD < 2465 \\ -4.45 & if \geq 2465 \end{cases}$$

Results of the analysis of weather options 4:

Figure 8 – Graph Illustrated payoff from the weather Options 4, 5 and 6 proposed for Nariño: The author´s Processing

- If the HDD index value at the maturity date of options is lower than 2220.07, then the hedger is profitable;
- Otherwise, he suffers a financial loss 4.93.



And the payoff of weather option 5 and 6 is performance as weather option 4.

We can deduce following conclusions. By buying put option or call option, the farmer establishes the highest payoff in the case of temperature decrease or increase. The cost of this benefit is that the buying put option does not allow the farmer to participate on the price increase and vice versa the buying call option does not allow the farmer to participate on the price decrease.

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